The Nervous System Appears to Minimize a Weighted Sum of Kinematic Error, Force, and Change in Force when Adapting to Viscous Environments during Reaching and Stepping

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The evolution of kinematic performance during reaching and walking in a viscous force field is well captured by a difference equation that relates the current and previous force to kinematic errors:

$$x_{i+1} = a_0 x_i + b_1 F_i + b_0 F_{i+1} + c_0 \qquad (1)$$

where x_i is a scalar measure of kinematic performance on the i^{th} movement (e.g. the average deviation from the normative trajectory), and F_i is a scalar measure of the force generated by the viscous force field (e.g. the average lateral force applied) (Fig. 1) [1-4]. These dynamics are consistent with the formation of an internal model of the force field [1, 2]. <u>Here, we prove that these same performance dynamics minimize a cost function *J* that is a weighted sum of kinematic error, force, and change in force:</u>

$$J = \frac{1}{2}(x_{i+1} - x_d)^2 + \frac{\lambda}{2}(u_{i+1} - \alpha u_i)^2 \qquad (2)$$

where u_i = the force from the arm on the i^{th} movement, and x_d = the desired kinematic performance. The parameter λ weights the cost of the kinematic error versus the force terms. The parameter α weights the cost of the force magnitude term versus the force change term, with $\alpha = 0$ weighting only magnitude, and $\alpha = 1$ weighting only change. The minimum of this cost function occurs when:

$$\frac{\partial J}{\partial u_{i+1}} = (x_{i+1} - x_d)\frac{\partial x_{i+1}}{\partial u_{i+1}} + \lambda(u_{i+1} - \alpha u_i) = 0 \qquad (3)$$

The robot/limb dynamics are spring dynamics for the class of viscous force field presumed here (Fig. 2):

$$x_i - x_d = \frac{1}{K}(F_i + u_i) \qquad (4)$$

where K = limb stiffness. The controller that minimizes the cost function is thus:

$$u_{i+1} = \alpha u_i - \frac{1}{\lambda K}(x_{i+1} - x_d) \qquad (5)$$

The next kinematic performance x_{i+1} can be estimated using a Taylor's series expansion of (4):

$$x_{i+1} = x_i + \frac{\partial x_i}{\partial u_i}\Delta u + \frac{\partial x_i}{\partial F_i}\Delta F = x_i + \frac{1}{K}(u_{i+1} - u_i) \qquad (6)$$

where we have assumed that the force field does not change, i.e. $\Delta F = 0$. Combining (5) and (6) gives:

$$u_{i+1} = f u_i - g(x_i - x_d) \qquad (7) \qquad f = \frac{\lambda\alpha K^2 + 1}{\lambda K^2 + 1} \qquad g = \frac{K}{\lambda K^2 + 1} \qquad (8)$$

The error-based learning controller (7) increments the motor command in proportion to the previous error, in the direction that reduces error. This controller can be viewed as implementing an inverse model D^{-1} that estimates the arm force u_{i+1} required for the desired kinematic performance x_d, i.e. $u_{i+1} = D^{-1}(x_d)$, where D^{-1} depends on experience (u_i and x_i). If $f < 1$ the controller increments a decremented version of the previous motor command in a "forgetting" process. The condition $f < 1$ corresponds to $\alpha < 1$ in the cost function. Thus, forgetting corresponds to minimizing force. This learning law is similar to recently proposed learning models [2, 5], except for the critical difference of the forgetting factor. Combining the controller (7) with the plant dynamics (4) gives:

$$x_{i+1} = (f - \frac{g}{K})x_i - \frac{f}{K}F_i + \frac{1}{K}F_{i+1} + (1 - (f - \frac{g}{K}))x_d \qquad (9)$$

i.e. the same dynamics as (1)

Thus, the controller that creates the learning dynamics (1) is an error-based learning law that constructs an internal model and incorporates a forgetting factor. This controller minimizes a weighted of sum of error, force, and change in force. The learning law parameters can be identified by multiple linear regression of (9) on experimental data. The cost function weights that best explain the data variance can be inferred from the learning law parameters using (8). For stepping in a viscous force field (Fig. 1), the identified forgetting factor was significantly less than one, ($p < 0.001$, mean = 0.77 ± 0.1 SD, 10 subjects) verifying that the CNS minimizes force per (2). A useful prediction based on these learning dynamics is that internal model formation can be accelerated by transiently amplifying the force field strength (Fig. 3) [4].

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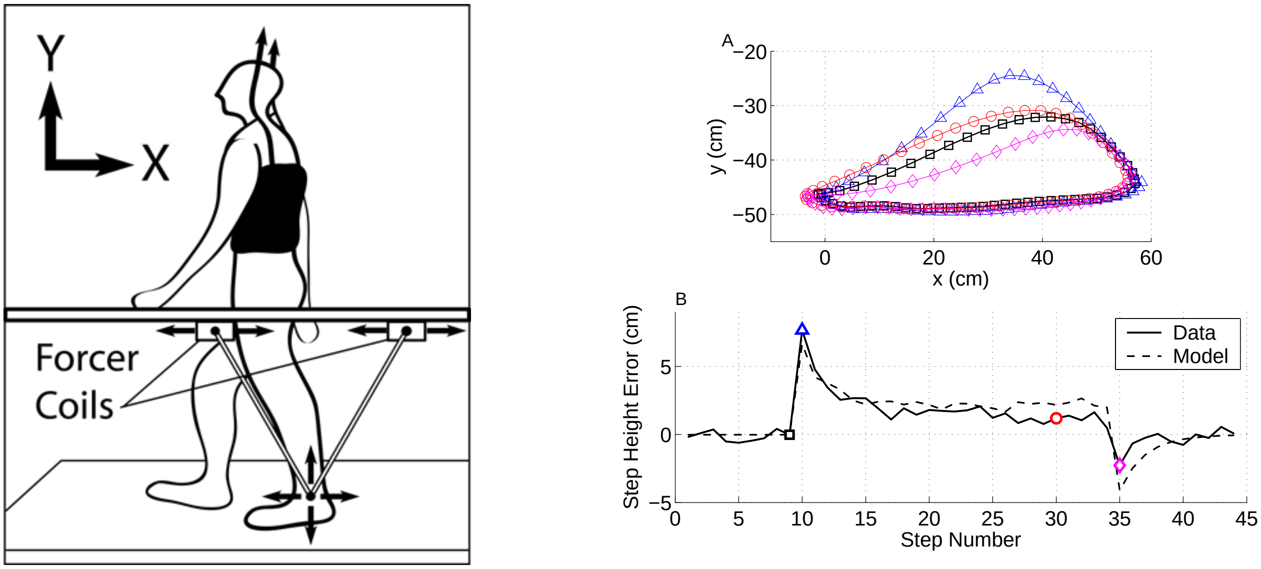

Figure 1 Left: Diagram of robotic device used to measure and perturb stepping. The robot makes use of a linear motor with two forcer coils and a V-shaped linkage to drive motion of its apex in the parasagittal plane. The apex is attached through a padded cuff and revolute joint to the subject's lower shank. We used the robot to apply an upward force field that had a magnitude proportional to the forward velocity of the leg during swing. **Right: Sample step data from a single subject for a single exposure to the force field.** A: Step trajectories of the lower shank during exposure to the force field. Shown are the normal stepping trajectory in the null field (squares); the "direct effect", which is the first step in force field (triangles); a step produced after adaptation to field exposure (circles); and the "after effect," which is the first step in the null field following adaptation (diamonds). B: Step height error, referenced to the normal stepping height, before, during, and after application of the force field. The force field was turned "on" at step 10 and "off" at step 35. The symbols correspond to the trajectories in A. The dashed line shows the best fit of Equation 1. For this subject, the fit produced $R^2 = 0.86$.

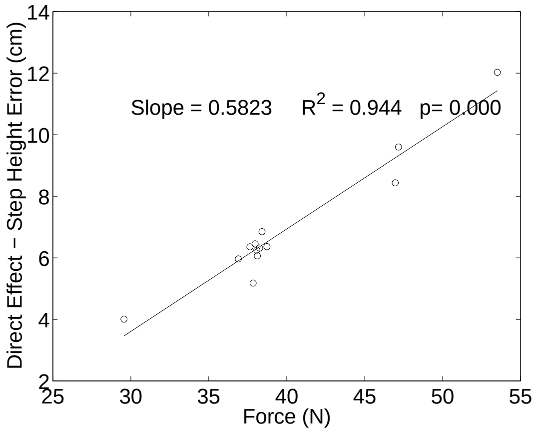

Figure 2 Spring-like dynamics of the leg in response to the viscous force field. Each data point is the change in step height due to unexpected application of the field. The field strength was varied; the abscissa shows the peak force applied. Each point is the average across ten subjects.

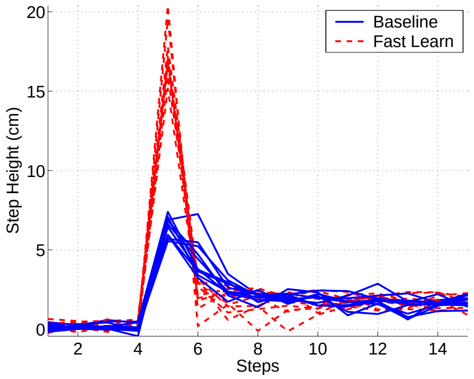

Figure 3 Example of how the learning dynamics can be exploited to accelerate learning. The "Baseline" data shows adaptation to the viscous force field during stepping for one subject, for ten exposures to the field (exposures overlaid, field turned on at step 5). By transiently amplifying the force field on Step 5 with a magnitude predictable using Equation (1), adaptation can be accelerated ("Fast Learn" experiment).